Statistics

Lecture 9

Feb 19-8:47 AM

Suppose I flip a loaded coin 10 times. (SG 16)

P(Tails) = .6   Success is to land tails.

$n = 10$ $p = .6$ $q = 1 - p = .4$

$np = 10(.6) = 6$ $npq = 10(.6)(.4) = 2.4$

$\sqrt{npq} = \sqrt{2.4} \approx 1.549$ $10 - 7 = 3$

P(exactly 7 tails) = $P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

$P(x=7) = {}_{10}C_7 \cdot (.6)^7 \cdot (.4)^3 \approx \boxed{.215}$

Using TI Command

2nd VARS ↓↓↓ Binompdf(

n → Trials: 10
p: .6
x-Value: 7
Paste Enter

No Menu
n p x
10, .6, 7
Enter

Your Work

$P(X=7) =$
binompdf(10,.6,7) =
.215

Jan 23-4:31 PM

P(at most 7 tails)

$P(x \leq 7) = P(x=7) + P(x=6) + P(x=5) + \cdots + P(x=0)$

$= \text{binomcdf}(\overset{n}{10}, \overset{p}{.6}, \overset{x}{7}) = \boxed{.833}$

P(fewer than 7 tails)

$P(x < 7) = P(x \leq 6) = \text{binomcdf}(10, .6, 6)$

6 7

$= \boxed{.618}$

P(atleast 5 tails) Total prob.

$P(x \geq 5) = 1 - P(x \leq 4) = 1 - \text{binomcdf}(10, .6, 4)$

we don't want 4 5 we want

$= \boxed{.834}$

Jan 23-4:41 PM

Miki is making random guesses on a True/False exam with 100 questions.

Success is to guess Correctly.

$n = 100$ $p = .5$ $q = .5$

$np = 100(.5) = \boxed{50}$ $npq = 100(.5)(.5) = \boxed{25}$ $\sqrt{npq} = \sqrt{25} = 5$

P(She gets 45 Correct answers) =

$P(x = 45) = \text{Binompdf}(100, .5, 45) = \boxed{.048}$

P(she gets at most 50 correct answers) =

$P(x \leq 50) = \text{binomcdf}(100, .5, 50) = \boxed{.540}$

P(She gets at least 40 Correct answers) =

$P(x \geq 40) = 1 - P(x \leq 39) = 1 - \text{binomcdf}(100, .5, 39)$

we don't want 39 40 we want

$= \boxed{.982}$

Total Prob.

SG 16 Page 4

use exact Value . . .

Jan 23-4:51 PM

Larissa is making random guesses on a multiple-choice exam with 120 questions. Each question has 5 choices with only one correct choice. Success is to guess correct Ans.

$n = 120$ $\quad p = \frac{1}{5} = .2$ $\quad q = \frac{4}{5} = .8$

$np = 120(.2) = \boxed{24}$ $\quad npq = 120(.2)(.8) = \boxed{19.2}$ $\quad \sqrt{npq} = \sqrt{19.2} \approx \boxed{4.382}$

P(she guesses exactly 30 questions correctly)

$P(x = 30) = \text{binompdf}(120, .2, 30) = \boxed{.035}$

P(she guesses fewer than 30 questions)

$P(x < 30) = P(x \leq 29) = \text{binomcdf}(120, .2, 29) = \boxed{.893}$

Jan 23-5:02 PM

P(she guesses at least 25 correct answers)

Total Prob.

$P(x \geq 25) = 1 - P(x \leq 24) = 1 - \text{binomcdf}(120, .2, 24)$

we don't want 24 | 25 we want

$= \boxed{.446}$

P(she guesses more than 20 correct answers)

$P(x > 20) = P(x \geq 21) = 1 - P(x \leq 20)$

$= 1 - \text{binomcdf}(120, .2, 20)$

we don't want 20 | 21 we want

$= \boxed{.785}$

Jan 23-5:10 PM

I randomly selected 400 voters.

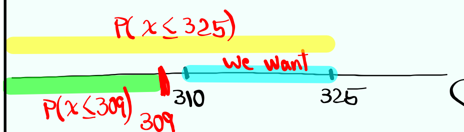
Prob. that any voter is in support of certain law is .8.

$n = 400$ $\quad p = .8$ $\quad q = .2$

$np = 320$ $\quad npq = 64$ $\quad \sqrt{npq} = 8$

P($\boxed{\text{\# of voters}}$ support the law is between 310 and 325, inclusive)

$P(310 \le x \le 325)$



$= \boxed{.656}$

$= P(x \le 325) - P(x \le 309)$

$=$ binomcdf(400, .8, 325) $-$ binomcdf(400, .8, 309)

Jan 23-5:17 PM

Erick flips a fair coin 800 times.

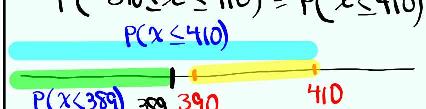
Success is to land tails.

1) $n = 800$ $\quad$ 2) $p = .5$ $\quad$ 3) $q = .5$

4) $np = 400$ $\quad$ 5) $npq = 200$ $\quad$ 6) $\sqrt{npq} \approx 14.142$

7) P(he gets between 390 and 410 tails, inclusive)

$P(390 \le x \le 410) = P(x \le 410) - P(x \le 389)$



Make sure

$=$ binomcdf(800, .5, 410) $-$ binomcdf(800, .5, 389)

$= \boxed{.542}$

Jan 23-5:26 PM

Mean μ

Variance σ^2

Standard deviation σ

$$\boxed{\begin{array}{l}\mu = np \\ \sigma^2 = npq \\ \sigma = \sqrt{\sigma^2}\end{array}}$$

Binomial Prob. Dist.

Suppose $n = 180$ $P = .4$, Binomial Prob. dist.

$q = 1 - P = \boxed{.6}$ $\mu = np = 180(.4) = \boxed{72}$

$\sigma^2 = npq = 180(.4)(.6) = \boxed{43.2}$

$\sigma = \sqrt{\sigma^2} = \sqrt{43.2} \approx \boxed{6.573}$

$\mu = 72$, $\sigma \approx 7$

68% Range $\Rightarrow \mu \pm \sigma = 72 \pm 7 \Rightarrow \boxed{65 \text{ to } 79}$

95% Range $\Rightarrow \mu \pm 2\sigma = 72 \pm 2(7) \Rightarrow \boxed{58 \text{ to } 86}$

Usual Range

Jan 23-5:48 PM

100 questions True/False only

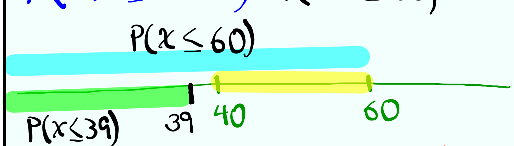
$n = 100$ $P = .5$ $q = .5$

$\mu = np = \boxed{50}$ $\sigma^2 = npq = \boxed{25}$ $\sigma = \sqrt{\sigma^2} = \boxed{5}$

Usual Range $\mu \pm 2\sigma = 50 \pm 2(5) \Rightarrow \boxed{40 \text{ to } 60}$

95% Range

$P(40 \le x \le 60) = P(x \le 60) - P(x \le 39)$



$\approx 96.5\%$

$\Rightarrow = \boxed{.965}$

$= \text{binomcdf}(100, .5, 60) - \text{binomcdf}(100, .5, 39)$

Jan 23-5:55 PM

Consider a binomial Prob. dist. with $n=250$ and $p=.6$.

1) $q = \boxed{.4}$

2) $\mu = 250(.6) = \boxed{150}$

3) $\sigma^2 = 250(.6)(.4) = \boxed{60}$

4) $\sigma = \sqrt{60} \approx 8$

5) Find 99.7% Range $= \mu \pm 3\sigma$

$= 150 \pm 3(8)$

$= 150 \pm 24 \Rightarrow \boxed{126 \text{ to } 174}$

6) $P(126 \leq x \leq 174)$

$= P(x \leq 174) - P(x \leq 125)$

$=$ binomcdf(250, .6, 174) − binomcdf(250, .6, 125)

$= \boxed{.998}$ 99.8%

SG 16

Jan 23-6:02 PM

Working with Continuous random Variable with prob. dist.

SG 17

SG 18

SG 19

SG 20

1) Uniform Prob. dist.

2) Standard normal prob. dist.

3) Normal Prob. dist.

4) Central Limit Theorem (CLT)

5) Applications

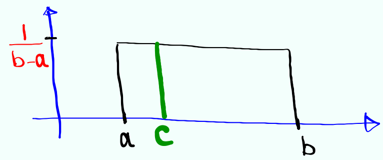
Jan 23-6:11 PM

Uniform Prob. Dist.:

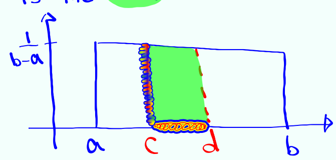
Let x be a continuous random variable for all values $\boxed{\text{from } a \text{ to } b}$ with uniform Prob. dist. $a \le x \le b$

1) $P(x=c)=0$ line has 0 area.

2) Graph is rectangular as shown below:



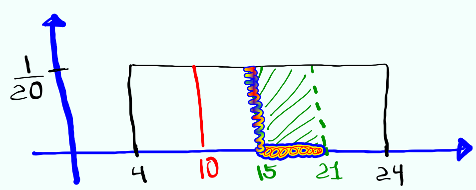
3) $P(c<x<d)$ is the area within the rectangle.



$$P(c<x<d) = (d-c)\cdot\frac{1}{b-a}$$

Jan 23-6:15 PM

ex. Consider a uniform Prob. dist. for all values from 4 to 24.



$P(x=10)=0$

Line has 0 area

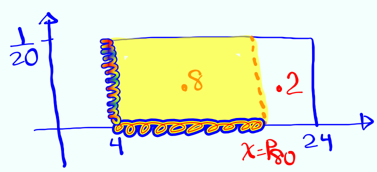
$P(15<x<21)$

$=(21-15)\cdot\frac{1}{20}=\frac{6}{20}=\frac{3}{10}=.3$

Find P_{80}

80% below

20% above



$(x-4)\cdot\frac{1}{20}=.8$

$x-4=20(.8)$

$x-4=16$

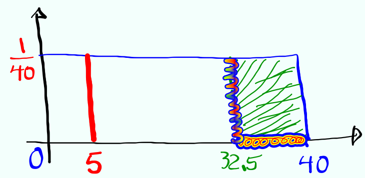
$\boxed{x=20}$

$P_{80}=20$

Jan 23-6:22 PM

Consider a uniform Prob. dist. for all values from 0 to 40.

1) Draw & clearly label.



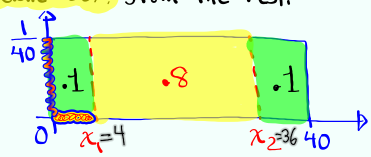
2) $P(x=5)=0$

3) $P(x>32.5)=(40-32.5)\cdot\frac{1}{40}$

$=\frac{7.5}{40}=\boxed{\frac{3}{16}}$

4) Find two values that separate the middle 80% from the rest.

$\frac{1-.8}{2}=.1$



$(x_1-0)\cdot\frac{1}{40}=.1$

$x_1-0=40(.1)$

$\boxed{x_1=4}$

$(40-x_2)\cdot\frac{1}{40}=.1$

$40-x_2=40(.1)$

$40-x_2=4$ $\boxed{x_2=36}$

Jan 23-6:30 PM

Standard Normal Prob. Dist.:

1) Use Z, $P(Z=c)=0$, Always use 3-decimal places.

2) Data dist. is symmetric, bell-shape with total area 1.

3) Mean = Mode = Median

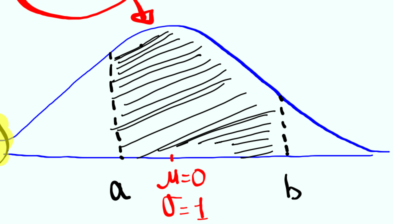
4) $\mu=0$ & $\sigma=1$

5) $\boxed{P(a<Z<b)}$ is the area of the corresponding region within the bell-curve.

How to find it:

2nd VARS

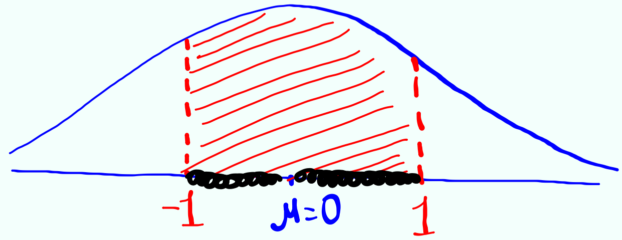
normalcdf(L, U, μ, σ)



Jan 23-6:42 PM

find $P(-1 < Z < 1)$

Drawing, Labeling, Shading, and Full TI Command required.



$= \text{normalcdf}(-1, 1, 0, 1)$

(–)

$= \boxed{.683}$

$\approx 68\%$

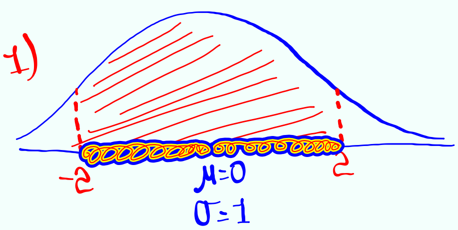
Jan 23-6:49 PM

$P(-2 < Z < 2)$

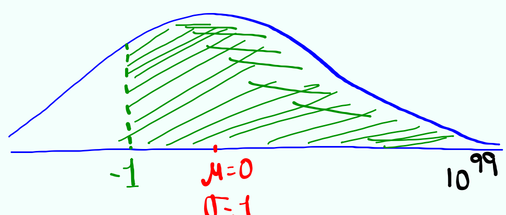
$= \text{normalcdf}(-2, 2, 0, 1)$

$= \boxed{.954}$

$\approx 95\%$



$P(Z > -1)$



$E99$

S.N.

2nd ,

$= \text{normalcdf}(-1, E99, 0, 1)$

(–)

2nd ,

$= \boxed{.841}$

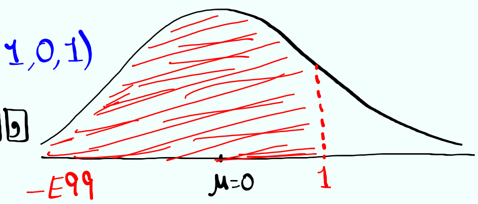
Jan 23-6:54 PM

$P(Z < 1)$

$= \text{normalcdf}(-E99, 1, 0, 1)$

(−) $\boxed{\text{2nd}}\boxed{9}$

$= \boxed{.841}$

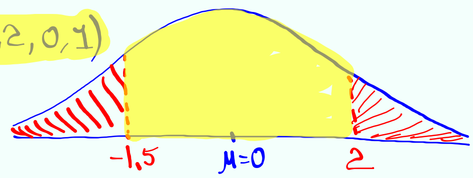


Find $P(Z < -1.5 \ \boxed{\text{OR}} \ Z > 2)$

$= 1 - \text{normalcdf}(-1.5, 2, 0, 1)$

Total Area

$= \boxed{.090}$



$P(Z < -1 \text{ and } Z > 2) = 0$

M.E.E.

Jan 23-7:01 PM

Find $Z = P_{90}$

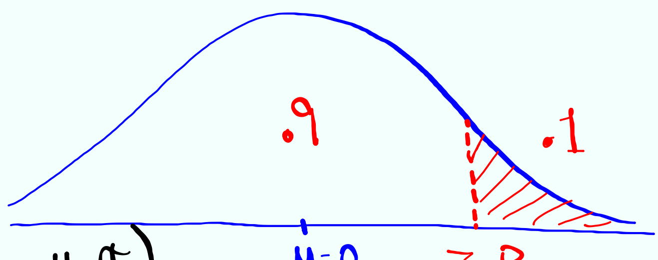
90% below

10% above

Doing Reverse

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

$\text{invNorm}(\text{Left Area}, \mu, \sigma)$



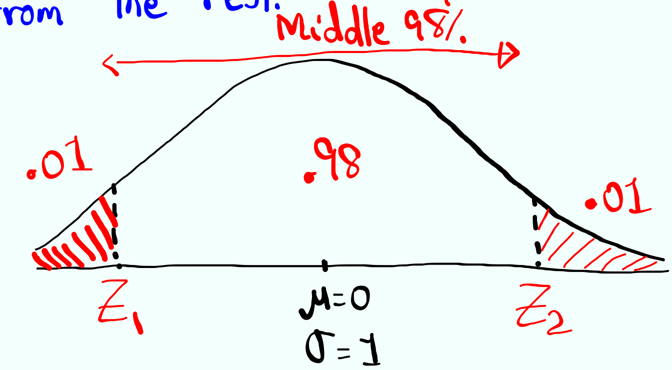
$Z = P_{90} = \text{invNorm}(.9, 0, 1)$

$\approx \boxed{1.282}$

Jan 23-7:11 PM

Find two Z-values that separate the middle 98% from the rest.

$$\frac{1-.98}{2}=.01$$



$Z_1=\text{invNorm}(.01,0,1)\approx\boxed{-2.326}$

By Symmetry

$Z_2=\text{invNorm}(.99,0,1)\approx\boxed{2.326}$

Jan 23-7:16 PM